Published in **AMESA-2 Proceedings**, Peninsula Technikon,
1-5 July 1996, 345-350.

An Interesting Duality in Geometry (Continued1)

Michael de Villiers, Mathematics Education
University of Durban-Westville

profmd@mweb.co.za

http://mzone.mweb.co.za/residents/profmd/homepage.html

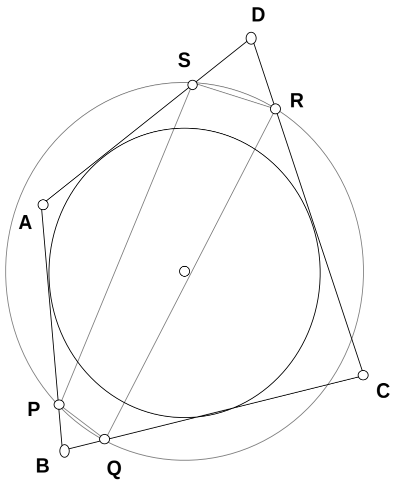

Figure 4

Theorem 1

Consider a convex circum quad as shown in Figure 4 with side lengths of AB=a, BC=b, CD=c and DA=d. Select *any* point P on AB. Take Q on BC so that BQ = PB, R on CD so that CR = QC and S on AD so that DS = RD. Then we have the surprising result that AS = AP and PQRS is a cyclic quadrilateral. (Note that this result is a generalization of the result that the four points where the incircle touches the sides of a circum quad are concyclic).

Proof

Let PA = x, then PB = $a - x$ = BQ, QC = $b - a + x$ = CR and RD = $c - b + a - x = (a + c) - b - x = (b + d) - b - x = d - x$ = SD ... ($a + c = b + d$ for a circum quad). Then AS = $d - (d - x) = x$.

Further note that all the angle bisectors of ABCD coincide with the perpendicular bisectors of the sides of PQRS (eg. consider isosceles Δ's ASP, BPQ, CQR and DRS). Thus the perpendicular bisectors of the sides of PQRS are concurrent (at the incentre of ABCD) and it is a cyclic quad. (Also note that for a fixed incentre, all the circumcircles arising from different choices of P will be *concentric* with the incircle of ABCD). QED.