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An Interesting Duality in Geometry (Continued2)

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Theorem 2

Construct any *angle divider* AQ of $\angle A$ of a convex *cyclic quad* ABCD. Now construct the angle divider BS of $\angle B$ so that $\angle PBA = \angle PAB$, the angle divider CR of $\angle C$ so that $\angle SCB = \angle SBC$ and the angle divider DQ of $\angle D$ so that $\angle RDC = \angle RCD$ (see Figure 5). Then $\angle QDA = \angle QAD$ and PQRS is a *circum quad*.

Proof

Let $\angle PAB = x$, then $\angle PBA = x$, $\angle SBC = \angle B - x = \angle SCB$, $\angle RCD = \angle C + x - \angle B = \angle RDC$ and
$\angle QDA = \angle D - \angle C - x + \angle B = (\angle D - \angle C + \angle B) - x = \angle A - x = \angle QAD$.

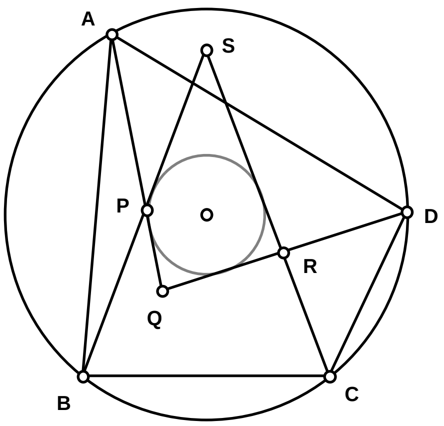

Figure 5

Further note that all the perpendicular bisectors of the sides of ABCD coincide with the angle bisectors of PQRS (e.g. consider isosceles Δ's PAB, SBC, RCD and QDA). Thus the angle bisectors of PQRS are concurrent (at the circumcentre of ABCD), and is it a circum quad. (Also note for a fixed circumcentre, that all the incircles arising from different choices of P will be *concentric* with the circumcircle of ABCD). QED.