In the September 1995 issue of the *Mathematics and Informatics Quarterly*, the following interesting result was mentioned without proof in the column on Forgotten Theorems:

**Kosnita's Theorem.** The lines joining the vertices $A$, $B$, and $C$ of a given triangle $ABC$ with the circumcenters of the triangles $BCO$, $CAO$, and $ABO$ ($O$ is the circumcenter of $ΔABC$), respectively, are concurrent.

On the basis of an often observed (but not generally true) duality between circumcenters and incenters (e.g. see De Villiers, 1996), I immediately conjectured that the following dual to Kosnita's theorem might be true, namely:

**Kosnita Dual.** The lines joining the vertices $A$, $B$, and $C$ of a given triangle $ABC$ with the incenters of the triangles $BCO$, $CAO$, and $ABO$ ($O$ is the incenter of $ΔABC$), respectively, are concurrent.

Investigation on the dynamic geometry program *Sketchpad* then confirmed that the conjecture was indeed true. I then found that both results could easily be proved by the following useful result for concurrency, a proof of which is given in De Villiers (1996). Interestingly, the point of concurrency for the special case with equilateral triangles on the sides is called the Fermat-Torricelli point.

**Fermat-Torricelli Generalization.** If triangles $DBA$, $ECB$ and $FAC$ are constructed on the sides of any triangle $ABC$ so that $∠DAB = ∠CAF$, $∠DBA = ∠CBE$ and $∠ECB = ∠ACF$ then $DC$, $EA$ and $FB$ are concurrent (see Figure 1).
It should further be pointed out that the above result is also true when the triangles are constructed inwardly. As shown in Figure 2 we clearly have for Kosnita's dual that $\angle DAB = \frac{1}{4} \angle A = \angle CAF$, $\angle DBA = \frac{1}{4} \angle B = \angle CBE$ and $\angle ECB = \frac{1}{4} \angle C = \angle ACF$, and from the above result it therefore follows that $DC, EA$ and $FB$ are concurrent.

![Figure 2](image1)

![Figure 3](image2)
Kosnita's theorem follows a little less directly from the Fermat-Torricelli generalization. In this case, the "base triangle" is triangle $DEF$ with $A$, $B$ and $C$ the outer vertices (see Figure 3). Since $DBOA$ is a kite, we have $\angle BDO = \angle ADO$. But $DBOE$ and $DOFA$ are also kites. Therefore, $\angle BDE = \frac{1}{2} \angle BDO$ and $\angle ADF = \frac{1}{2} \angle ADO$ from which follows that $\angle BDE = \angle ADF$. In a similar fashion can be shown that $\angle BED = \angle CEF$ and $\angle CFE = \angle AFD$. From the F-T generalization, it therefore follows that $DC$, $EA$ and $FB$ are concurrent.

Reference