

## Olympic Mathematics: Is it fair?

Michael de Villiers, Mathematics Education, UDW

profmd@mweb.co.za

<http://mzone.mweb.co.za/residents/profmd/homepage.html>

At the 1996 Atlanta Olympic Games Merlene Ottey (Jamaica) and Gail Devers (USA) clocked exactly the same time for the Women's 100m, namely, 10.88 s. But Gail Devers was awarded the gold medal, despite the fact that Ottey caught up with her in the latter stage of the race and would have overtaken her had the race been only a meter longer.

Was this a fair decision? On what basis was this decision made?

According to the commentators, the Olympic times are actually measured to a thousandth of a second, and according to those times Devers was faster. So apparently it was fair. Or was it?

Suppose the time difference was 0.001 s, what was the minimum possible distance between the two athletes? To simplify matters, let's assume that both athletes maintain the same constant speed throughout. The slowest time Devers could therefore have run was 10.883s with Ottey at 10.884 s. (If Ottey had run 10.885, her time would've been given as 10.89). So Ottey's average speed was  $\frac{100}{10.884}$  m/s. The distance she would have covered in 10.883s would therefore have  $\frac{100}{10.884} \times 10.883 = 99.9908$  m. In other words, Devers would have been approximately 9.2 mm ahead of Ottey.

Furthermore, if we take into account that the times of 10.883 s and 10.884 s themselves are rounded off, their actual times to four decimals could, for example, respectively have been 10.8834 s for Devers and 10.8835 s for Ottey. If this were the case, the shortest possible distance between them would have been only about:

$$100 - \frac{100}{10.8835} \times 10.8834 = 0.0009\text{m} = 0.9 \text{ mm.}$$

Now this really is getting to the point of splitting hairs, not figuratively, but literally! A difference of 0.9 mm could be the difference in thickness of their vests or the lengths of the lanes they are running in. Therefore, if the time difference was only 0.001 s

it would seem more fair to have given gold medals to both athletes.

The *minimum possible* distances ( $\Delta d$ ) for a few time differences ( $\Delta t$ ) are given in Table 1.

$\Delta t$ (in s)	$\Delta d$ (in cm)
0.001	0.09
0.002	1.01
0.003	1.93
0.004	2.85
0.005	3.77

**Table 1**

So it would appear that only from about a time difference of 0.003 s, the minimum possible distance between them becomes significant.

Interestingly, in longer races or slower events it makes even less sense to measure time differences to one thousandth of a second. Consider for example a 100 m breaststroke swimming race where two swimmers clock the same time of say, 62.11 s. For a 0.001 s time difference, the slowest possible time of the winner would be 62.113 s and that of the loser 62.114 s. Since these numbers are themselves rounded off, the actual times to four decimals could, for example, respectively have been 62.1134 s and 62.1135 s. This gives a minimum possible distance of

$$100 - \frac{100}{62.1135} \times 62.1134 = 0.0002 \text{ m} = 0.2 \text{ mm.}$$

The reason is simply that for larger numbers (slower times) a difference of 0.001 is relatively smaller in comparison to the size of the numbers than for smaller numbers (faster times).

$\Delta t$ (in s)	$\Delta d$ (in cm)
0.001	0.02
0.002	0.18
0.003	0.34
0.004	0.50

0.005	0.66
0.006	0.82
0.007	0.98
0.008	1.14
0.009	1.30

**Table 2**

For the sake of comparison, the minimum possible distances ( $\Delta d$ ) for a few time differences ( $\Delta t$ ) for the swimming race are given in Table 2. As shown in the table, even for a time difference of 0.009 s, the minimum possible distance between them would not be very significant.

One cannot help but wonder whether athletes and sports administrators are aware of mathematical considerations like those given above.

XXXXXXXXXXXXXXXXXXXXXXXXXXXX

*"The use of applied mathematics in relation to a physical problem involves three stages: (1) a dive from the world of reality into the world of mathematics; (2) a swim in the world of mathematics; (3) a climb back into the world of reality, carrying a prediction in our teeth."*

- A. G. Heaton